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Ion-acoustic Double Layer in a Multicomponent Plasma with Two Types of Positive Ions and Two-temperature Isothermal Electrons

Basudev Ghosh[1*], Sreyasi Banerjee[2]
1. Department of Physics, Jadavpur University, Kolkata-700032, India.
2. Department of Electronics, Vidyasagar College,Kolkata, India
*Corresponding author: Basudev Ghosh, E-mail: bsdvghosh@gmail.com
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ABSTRACT

Using integral form of governing equations in terms of pseudo potential ion-acoustic double-layers have been investigated theoretically in a multicomponent plasma consisting of two types of cold positive ions and two- temperature isothermal electrons. The mathematical technique used here has advantage over other conventional methods to study higher order nonlinear and dispersive effects on double layer formation. Conditions are obtained for the existence of ion-acoustic double-layers in the studied plasma. The effects of temperature and density ratios of the two types of electrons and the ratio of masses of the two types of positive ions on the double-layer formation and structure are also investigated. They are shown to have significant effects on the excitation and structure of the double-layers. The importance of this investigation has also been pointed out.

Keywords: Double layer; Multicomponent plasma; Two-temperature electrons

INTRODUCTION

A double layer (DL) in plasma consists of two oppositely charged parallel layers resulting in a strong electric field across the layer which can accelerate the plasma electrons and ions in opposite directions. DLs occur naturally in a variety of space plasma environments [1-4] and are of considerable interest in astrophysics [5-9]. Various theories on the formation of solar flares also involve DLs [5].The Viking satellite [1] observations show the presence of DLs in the magnetospheric regions. The S3-3 satellite [4] observations show the presence of DLs in the auroral regions. It has also been discovered that the acoustic DLs are responsible for auroral electron precipitation. During the past two decades the formation of DLs has been a topic of great interest because of their relevance to cosmic applications [5-10], ion heating in linear turbulent heating devices [10] and confinement of plasma in tandem mirror devices [11].Using reductive perturbation technique several authors have studied weak ion-acoustic DLs in different plasma systems [12-18]. Space plasmas are of multispecies type and offer a rich source for studying DLs. In recent years there has been considerable interest in the study of ion-acoustic solitary waves and DLs in multispecies plasmas [19-23].

In some space environments and experimental situations plasmas can be found to have multicomponent composition with two-temperatures electrons. The presence of two-temperature electrons and several ion species in plasma are expected to give rise to many interesting characteristics in nonlinear propagation of waves including the excitation of ion-acoustic solitary waves and double layers in plasmas [7,8]. Nonlinear wave propagation in multicomponent plasma with two types of ions and two-temperature electrons has been studied by a number of authors [24-29]. It has been shown that a small percentage of the cooler component of electrons leads to effects qualitatively different from those obtained for a plasma having only one-temperature electron species.Most of these authors used standard reductive perturbation method which includes up to second order nonlinearity and dispersive effects. As the consideration of higher order terms is expected to give theoretical results more closer to the experimental findings, researchers have also attempted to consider the contribution of higher order nonlinear terms by using a pseudopotential technique [30,31].which uses integral form of governing equations in terms of pseudopotential. Recently this technique has been successfully used by some authors [22,32-34] to investigate the effects of higher order nonlinear and dispersive terms on the propagation of ion-acoustic solitary waves in different plasma systems.Very recently Ghosh et al[22] have considered the effects of higher order nonlinearity and dispersive effects on ion-acoustic solitary waves in plasmas with two positive ions and two-temperature isothermal electron by using the above pseudopotential technique. This method has an advantage over standard reductive perturbation method. Here instead of solving a second order inhomogeneous differential equation at each order one has to solve a first order inhomogeneous equation at each order except at the first. The purpose of the present paper is to investigate DLs in plasmas with two positive ions and two-temperature isothermal electron by using the above pseudopotential technique.

The paper is organized as follows: Starting from the basic equations we first derive the nonlinear evolution equation and then find solitary wave solutions at different orders. Next we consider the critical stage and finally discuss the results.

BASIC EQUATIONS

Let us consider a plasma consisting of two types of cold positive ions and two temperature isothermal electrons The plasma is assumed to be collisionless and unmagnetized and hence the nonlinear behaviour of ion-acoustic waves in such a plasma may be described by the following set of normalized basic equations:

$$\frac{\partial n_{is}}{\partial t}+\frac{\partial}{\partial x}(n_{is}v_{is})=0 \qquad (1)$$

$$\frac{\partial v_{is}}{\partial t}+v_{is}\frac{\partial v_{is}}{\partial x}+\mu_{is}\frac{\partial \phi}{\partial x}=0 \qquad (2)$$

$$\frac{\partial^2\phi}{\partial x^2}=n_{el}+n_{eh}-\sum_{s=1,2}n_{is} \qquad (3)$$

where s = 1, 2 stands for the two types of positive ions, v_{is} is the velocity of the ions having mass m_{is} and number density n_{is}. n_{el} and n_{eh} are the densities of low and high temperature electrons respectively. n_0 is the equilibrium number density of ions. k_B is the Boltzmann constant. ϕ is the electrostatic potential, $\mu_{is}=m_{i1}/m_{i2}$ is the mass ratio of two types of positive ions. T_{eff} is the effective temperature of the plasma defined by

$$T_{eff}=\frac{T_{el}.T_{eh}}{\mu T_{eh}+\nu T_{el}} \qquad (4)$$

where μ and ν are the unperturbed number densities of low and high temperature electrons respectively.

The charge neutrality condition in the plasma is given by

$$n_{i10}+n_{i20}=\mu+\nu=1 \qquad (5)$$

The isothermal electron densities are given by

$$n_{el}=\mu e^{\frac{\phi}{\mu+\nu\beta}} \qquad (6)$$

$$n_{eh}=\nu e^{\frac{\beta\phi}{(\mu+\nu\beta)}} \qquad (7)$$

where $\beta=T_{el}/T_{eh}$ is the ratio of temperatures of the two types of electrons.

In the above Eqs.(1)-(7) ions and electron densities are normalized with respect to the equilibrium number density of ions n_0, the distances are normalized by the Debye length $\lambda_D=\left(k_BT_{eff}/4\pi e^2n_0\right)^{1/2}$, time by the ion plasma period $\omega_{is}^{-1}=\left(m_{is}/4\pi e^2n_0\right)^{1/2}$, velocities by ion-acoustic speed $C_{is}=\left(k_BT_{eff}/m_{is}\right)^{-\frac{1}{2}}$ and potential ϕ by k_BT_{eff}/e.

In order to study time-independent DL structure we make all the dependent variables depend only on a single independent variable ξ, where $\xi=x-Vt$ in which V is the Mach number with respect to the ion-acoustic speed is the velocity of the solitary wave. In terms of ξ, Eq. (3) can be rewritten as

$$\frac{d^2\phi}{d\xi^2}=n_{el}+n_{eh}-\sum_{s=1,2}n_{is} \qquad (8)$$

Expressing Eqs.(1) and (2) in terms of ξ and using the boundary conditions

$n_{is}\to 1, \phi\to 0$ as $\xi\to\pm\infty$

we obtain

$$\sum_{s=1,2}n_{is}=\sum_{s=1,2}n_{s0}\left(1-\frac{2\mu_{is}}{V^2}\right)^{1/2} \qquad (9)$$

Substituting in Eq. (8) for η_{el}, η_{eh} and $\sum n_{is}$ respectively from (6), (7) and (9) we get,

$$\frac{d^2\phi}{d\xi^2}=\Delta_1\phi+\Delta_2\phi^2+\Delta_3\phi^3+\Delta_4\phi^4 \qquad (10)$$

where,

$$\Delta_1=1-\frac{\mu_{is}}{V^2}$$

$$\Delta_2=\frac{\mu+\nu\beta^2}{2(\mu+\nu\beta)^2}-\frac{3}{2}.\frac{\mu_{is}{}^2}{V^4}$$

$$\Delta_3=\frac{\mu+\nu\beta^3}{6(\mu+\nu\beta)^3}-\frac{5}{2}.\frac{\mu_{is}{}^3}{V^6}$$

$$\Delta_4=\frac{\mu+\nu\beta^4}{24(\mu+\nu\beta)^4}-\frac{35}{8}.\frac{\mu_{is}{}^4}{V^8} \qquad (11)$$

DOUBLE LAYER SOLUTION

We stretch the ξ -coordinate according to the relation

$$x=\varepsilon\cdot\xi \qquad (12)$$

where ε is a small parameter measuring the weakness of dispersion. Then Eq.(10) becomes

$$\varepsilon^2\frac{d^2\phi}{dx^2}=\Delta_1\phi+\Delta_2\phi^2+\Delta_3\phi^3+\Delta_4\phi^4 \qquad (13)$$

An integration of this equation under the conditions

$$\frac{d\phi}{dx}\to 0,\ \phi\to 0 \quad \text{as} \quad x\to\pm\infty \qquad (14)$$

gives the energy integral equation:

$$\frac{1}{2}\varepsilon^2\left(\frac{d\phi}{dx}\right)^2=\frac{1}{2}\Delta_1\phi^2+\frac{1}{3}\Delta_2\phi^3+\frac{1}{4}\Delta_3\phi^4+\frac{1}{5}\Delta_4\phi^5 \qquad (15)$$

In order to consider higher order nonlinearity we make an approximation by taking the coefficients of ϕ^3 in Eq. (15) to be of the order of ε i.e. we take the coefficient of ϕ^3 in Eq.(15) as $\frac{1}{3}\varepsilon\Delta_2$. We now make the following perturbation expansions for ϕ and V:

$$\phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \varepsilon^3\phi^{(3)} + \cdots \quad (16)$$

$$V = V_0 + \varepsilon^2 V^{(1)} + \varepsilon^3 V^{(2)} + \cdots \quad (17)$$

where $V_0 = (\mu_{is})^{\frac{1}{2}}$ is the linear velocity of the wave [22].

Using the expansion (17) in Eqs. (11) we obtain

$$\Delta_1 = \varepsilon^2\delta_1^{(1)} + \varepsilon^3\delta_1^{(2)} + \cdots$$
$$\Delta_2 = \delta_2^{(0)} + \varepsilon^2\delta_2^{(1)} + \varepsilon^3\delta_2^{(2)} + \cdots$$
$$\Delta_3 = \delta_3^{(0)} + \varepsilon^2\delta_3^{(1)} + \cdots$$
$$\Delta_4 = \delta_4^{(0)} + \cdots \quad (18)$$

where

$$\delta_2^{(0)} = \frac{\mu + v\beta^2}{2(\mu + v\beta)^2} - \frac{3}{2},\ \delta_2^{(1)} = \frac{6\mu_{is}V^{(1)}}{V_0^3},\ \delta_2^{(2)} = -\frac{6V^{(1)^2}}{V_0^3}$$

$$\delta_1^{(1)} = \frac{2V^{(1)}}{V_0}, \quad \delta_1^{(2)} = \frac{2\mu_{is}}{V_0^3}\left[V^{(2)} - \frac{3}{2}.\frac{V^{(1)^2}}{V_0}\right]$$

$$\delta_3^{(0)} = \frac{\mu + v\beta^3}{6(\mu + v\beta)^3} - \frac{5}{2}, \quad \delta_3^{(1)} = \frac{16V^{(1)}}{V_0} \quad (19)$$

$$\delta_4^{(0)} = \frac{\mu + v\beta^4}{24(\mu + v\beta)^4} - \frac{35}{8}$$

Substituting the expansions (16)-(19) in Eq. (15) we get a sequence of equations for $\phi^{(i)}$ and the equation for $\phi^{(i)}$ at each order becomes a first order inhomogeneous differential equation for *i* > 1.Equating coefficients of ε^4 on both sides of Eq. (15) we get the so-called energy equation in the lowest order as

$$\frac{1}{2}\left(\frac{d\phi^{(1)}}{dx}\right)^2 + \psi(\phi^{(1)}) = 0 \quad (20)$$

where

$$\psi(\phi^{(1)}) = -\frac{1}{2}\delta_1^{(1)}\phi^{(1)^2} - \frac{1}{3}\delta_2^{(0)}\phi^{(1)^3} - \frac{1}{4}\delta_3^{(0)}\phi^{(1)^4} \quad (21)$$

It is interesting to note that Eq. (20) may be interpreted as to describe as the one-dimensional motion of a pseudo particle of unit mass with velocity $d\phi^{(1)}/dx$ and position $\phi^{(1)}$ in a potential well $\psi(\phi^{(1)})$. The first term in Eq. (20) can be regarded as the kinetic energy of the pseudo particle and whereas the second term is the potential energy of the same particle at that instant.. Since kinetic energy is always a non-negative quantity $\psi(\phi^{(1)}) \le 0$ for the entire motion. Thus zero is the maximum value of $\psi(\phi^{(1)})$. We can consider $\psi'(\phi^{(1)})$ as the force acting on the particle at the position $\phi^{(1)}$. Equation (20) may also be imagined as an equation of anharmonic oscillator provided that we interpret $\phi^{(1)}$ and x as space and time coordinates respectively.

For double layer solution of Eq. (20) the potential $\psi(\phi^{(1)})$ should be negative between $\phi^{(1)} = 0$ and $\phi_m^{(1)}$ where $\phi_m^{(1)}$ is some extremum value of the potential $\phi^{(1)}$, called the amplitude of the Double-layer. For double layer solutions $\psi(\phi^{(1)})$ must satisfy the following additional boundary conditions:

i) $\psi(\phi^{(1)}) = 0, \psi'(\phi^{(1)}) = 0$ and $\psi''(\phi^{(1)}) < 0$ at $\phi^{(1)} = 0$ (22)

ii) $\psi(\phi^{(1)}) = 0, \psi'(\phi^{(1)}) = 0$ and $\psi''(\phi^{(1)}) < 0$ at $\phi^{(1)} = \phi_m^{(1)} (\neq 0)$. (23)

iii) $\psi(\phi^{(1)}) < 0$ for $0 < |\phi^{(1)}| < |\phi_m^{(1)}|$ (24)

When the above conditions are satisfied $\phi^{(1)} = 0$ is an unstable position of equilibrium. If the particle is slightly displaced from this unstable position of equilibrium, it moves

away from this unstable position of equilibrium and it continues its motion until its velocity

is equal to zero at some position $\phi_m^{(1)}$. The pseudo-particle is not reflected at $\phi^{(1)} = \phi_m^{(1)}$ because of simultaneous vanishing of pseudo force and pseudo velocity. Instead, it goes to another state producing an asymmetrical double layer with a net potential drop of $\phi_m^{(1)}$.

Applying boundary conditions (22), (23) we get from (21)

$$\phi_m^{(1)} = -\frac{2\delta_2^{(0)}}{3\delta_3^{(0)}} \quad (25)$$

and

$$\psi(\phi^{(1)}) = -\frac{\delta_3^{(0)}}{4}\phi^{(1)2}(\phi_m^{(1)} - \phi^{(1)})^2 \quad (26)$$

The DL solution of Eq. (20) using (26) is given by

$$\phi^{(1)} = \frac{\phi_m^{(1)}}{2}[1 - \tanh(\sqrt{\delta_3^{(0)}/8}.\phi_m^{(1)}.\mathrm{x})] \quad (27)$$

$\phi_m^{(1)}$ as given by Eq. (25) is the amplitude of the DL. It is to be noted that for the existence of a DL the coefficient $\delta_3^{(0)}$ of the cubic nonlinear term must be positive. This requires that for a DL solution plasma parameters must satisfy the following condition

$$\mu + v\beta^3 > 15(\mu + v\beta)^3 \quad (28).$$

From Eqs. (25) and (27) it is clear that the nature of DLs i.e. a compressive or rarefactive double-layer, depends on the sign of $\delta_2^{(0)}$. the coefficient of the quadratic nonlinear term. If $\delta_2^{(0)}$ is positive a rarefactive DL is formed. On the other hand if $\delta_2^{(0)}$ is negative a compressive DL is formed. The width of the DL is given by

$$w = 2\sqrt{\delta_3^{(0)}/8}\,/\left|\phi_m^{(1)}\right| \tag{29}$$

Note that the nature, amplitude and width of the DLs depend on plasma parameters such as the temperature ratio of cold and hot electrons and density ratio of cold and hot electrons through the coefficients $\delta_2^{(0)}$ and $\delta_3^{(0)}$.

It is to be noted from Eqs.(19) and (25) that the double layer amplitude $\phi_m^{(1)}$ in the lowest order is independent of the mass ratio μ_{is} of two positive ions. This indicates that the nature of DL (i.e. compressive or rarefactive) does not depend on the mass ratio of the the two positive ions. On the other hand the unperturbed number densities of two groups of electrons μ and ν and the temperature ratio β of the two groups of electrons control the nature and characteristics of the double layer. The nonlinear coefficient $\delta_2^{(0)}$ may be either positive or negative depending on the relative values of μ (unperturbed number density of cooler component of electron) and β (the ratio of temperature of cooler component of electron and hotter component of electron). Hence both compressive and rarefactive types of double layer can be formed in the model plasma under consideration. Keeping all other parameters fixed if we slowly increase the value of β the nature of the DL changes from compressional mode to rarefactive mode. The critical value of β at which the nature of the DL changes is given by $\delta_2^{(0)} = 0$.Thus we get a critical value of β given by

$$\beta_c = \frac{6\mu\nu \pm \sqrt{36\mu^2\nu^2 - 4\nu(1-3\nu).\mu(1-3\mu)}}{2\nu(1-3\nu)} \tag{30}$$

Similarly we find that there is a critical value of μ, the unperturbed number density of cooler component of electron, at which the nature of the DL changes.Equation (29) shows that the double layer width *w* is a function of double layer amplitude $\phi_m^{(1)}$.

RESULTS AND DISCUSSIONS

In this paper we have investigated the conditions of formation and structure of DL in a multicomponent plasma containing two cold positive ions and two-temperature isothermal electrons. Numerically we have shown that the formation of both compressive and rarefactive types of DL is possible in the model plasma under study in different parametric regions (Figs. 1 and 2). In Fig.1 we show the formation of rarefactive double layer. The amplitude of the rarefactive double layer is found to increase with increase in μ, the ratio of number densities of the low and high temperature groups of electrons. Fig 2 shows the formation of compressive double layer. It shows that amplitude of the compressive double layer decreases with increase in μ. In fact the unperturbed number densities of the two groups of electrons, μ and ν, and the temperature ratio, β, of the two groups of electrons control the formation, nature and characteristics of the double layer. There exist critical values of these parameters at which the nature of DL changes.

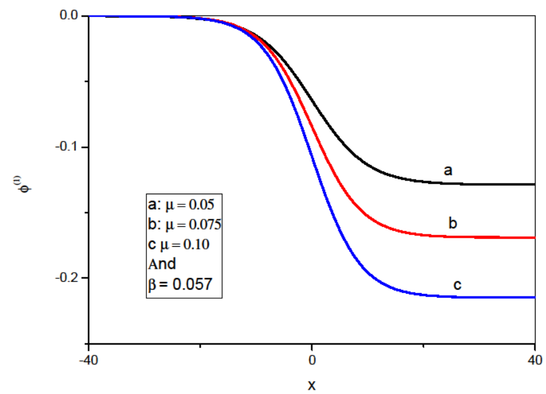

Fig. 1 Profile of rarefactive double layer for different values of μ

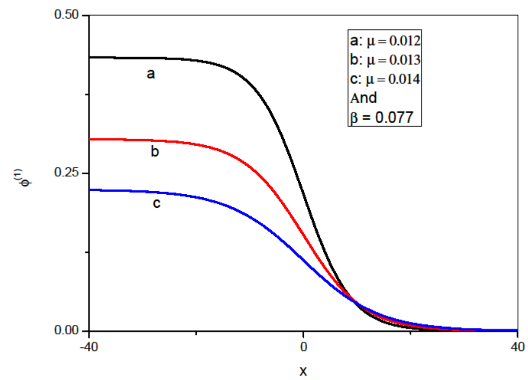

Fig. 2 Profile of compressive double layer for different values of μ

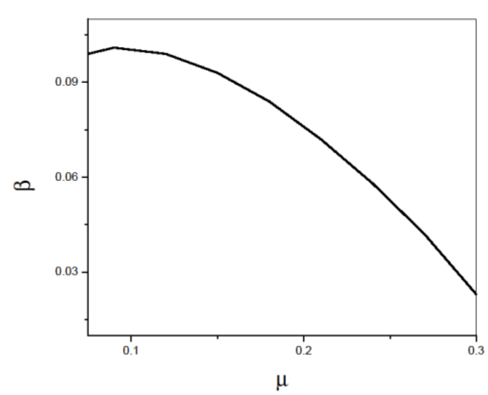

Fig. 3 Variation of the critical value of β with μ

We find that there is a critical value of β below which the DL is rarefactive in nature and above which the DL is compressive. Again this critical value β_c of β depends on the values of the ratio of number densities of the two groups of electrons. In Fig 3 we show how β_c

changes with the ratio of number densities of the two groups of electrons.

The double layer amplitude in the lowest order is found to be independent of the mass ratio μ_{is} of two positive ions. This indicates that the nature of DL (i.e. compressive or rarefactive) does not depend on the mass ratio of the two positive ions.

However if one goes to one order higher in nonlinearity and dispersion the mass ratio will begin to play important role in the structure and formation of DLs.

Finally we would like to point out that the results presented in the paper may be useful in the study of nonlinear wave phenomena in many space plasma environments. The mathematical technique used here has advantage over other conventional methods and may be adopted to study higher order nonlinear and dispersive effects on solitary waves in other plasma environments.

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